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NEW CONTRA MAPPINGS IN NANO TOPOLOGICAL SPACES

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ABSTRACT

Nano topology was initiated by M.Lellis Thivagar with regard to a subset X of a universe which is described in terms of lower, upper and boundary approximations of X. He also described nano interior and nano closure in nano topological spaces. In this paper, we define some new type of contra mappings namely contra Nano bc-continuous mapping in Nano topological spaces. In addition to this, we discussed some properties of contra nano bc-continuous mappings in nano topological spaces.

KEYWORDS: nano bc-open set, nano bc-continuous mapping, contra nano bc-continuous mapping.

I. INTRODUCTION

The study of *nano topology* was started by M. Lellis Thivagar et al.[8] with regard to a subset X of a universe that is described in terms of lower, upper and boundary approximations of X. He additionally described *nano interior* and *nano closure* in *nano topological spaces*.(or briefly №T Spaces). Andrijevic.[1] presented and studied a category of generalized open sets in a topological space referred to as b-open sets. Further C. Indirani et al.[4] created and studied *nano* b-open sets (№ bo sets) in *nano topological spaces* (№TS). B𝕔 open sets were first introduced in topological spaces by Hariwan Z. Ibrahim [6]. Here we proceed to present our findings on nano b$\mathbb{C}$-open mappings in Nano Topological Spaces.

II. PRELIMINARIES

Definition 2.1. [8] Let U denote a non-empty finite set of elements referred to as universe and R represents an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is called as the approximation space. Let X$\subseteq$ U.

(i) ***The lower approximation*** of X with respect to R is the set of all elements, which can be for certain classified as X with respect to Rand it is denoted by $L_{\mathcal{R}}$(X).That is, $L_{\mathcal{R}}(X)=\cup_{x\in U}\{R_x : R_x \subseteq X\}$ where R_x denotes the equivalence class determined by x $\in$U.

(ii) ***The upper approximation*** of X with respect to R is the set of all elements, which can be possibly classified as X with respect to R and it is denoted by$U_{\mathcal{R}}$(X). That is$U_{\mathcal{R}}(X) = \cup_{x\in U}\{R_x: R_x \cap X \neq \emptyset\}$.

(iii) ***The boundary region*** of X with respect to R is the set of all elements, that can be classified neither as X nor as not-X with respect to R and it is denoted by$B_{\mathcal{R}}$(X). That is, $B_R(X)=U_{\mathcal{R}}(X)-L_{\mathcal{R}}(X)$.

Definition 2.2. [8] Let U represent the universe and R represent an equivalence relation on U. Thenτ_R (X)=№^{T} = { U, $\emptyset$, $L_{\mathcal{R}}$(X),$U_{\mathcal{R}}$(X), $B_{\mathcal{R}}$(X)} where X$\subseteq$U.Then, $\tau_{\mathcal{R}}$(X)satisfies the axioms listed below.

(i) U and $\emptyset \in \tau_{\mathcal{R}}$(X).

(ii) The union of the elements of any sub collection of $\tau_{\mathcal{R}}$(X) is in $\tau_{\mathcal{R}}$(X).

(iii) The intersection of the elements of any finite subcollection of $\tau_{\mathcal{R}}$(X) is in $\tau_{\mathcal{R}}$(X).That is, $\tau_{\mathcal{R}}$(X)is a topology on U referred to as the *nano topology* (№^{T}) on U with respect to X. We call (U,$\tau_{\mathcal{R}}$(X)) (or) (U,№^{T})as the *nano topological space*(№TS-in short).The elements of№^{T} are known as № *open sets*(briefly,№-OS). The complement of №-open sets are №-*closed* sets(briefly, №-CS).

Example 2.3. [8] Let U={p,q,r,s} with U/R = {{p},{q},{r,s}}and X={p,r} $\subset$U. Then the nano topology is $\tau_{\mathcal{R}}(X)$ =№^{T}={ U, $\emptyset$,{p},{r,s},{p,r,s}}.

Remark 2.4. [8] If $\tau_{\mathcal{R}}$(X) =№^{T}is the nano topology on Uwith respect to X and B_N isanano subset of №TS,then B_N ={ U, $L_{\mathcal{R}}(X), B_{\mathcal{R}}(X)$} is referred to as the basis for $\tau_{\mathcal{R}}$(X) .

Definition 2.5 [8] If (U,№^{T}) is a №TS with respect to X where
X ⊆U and if A_N is a nano subset in №TS and if A_N ⊆ U, then
(1) The Nano interior of A_N is defined as the union of all nano-open subsets of A and it is denoted by №-int(A_N). That is, №-int(A_N) is the largest nano-open subset of A_N.
(2) The Nano closure of A_N is defined as the intersection of all nano closed sets containing A_N and it is denoted by №-cl(A_N). That is, №-cl(A_N) is the smallest nano closed set containing A_N.

Definition 2.6. Let (U, $\tau_{\mathcal{R}}(X)$) be a №TS and A_N ⊆ U. Then A_N is said to be

(1) Nano-semi open set (№-SO set) [8] if A_N⊆№-cl[№-int(A_N)] and **Nano semi-closed** (№-SC set) [7] if №- int [№-cl(A_N)] ⊆ A_N.

(2) Nano-θopen set (№-θO set) [3] if for each x∈A_N, there exists a nano open set (№-OS) G such that x ∈ G ⊂№cl(G)⊂A_N.

(3) Nano-θsemiopen (№-θSO) [3] if for each x ∈A_N ,there exists a nano semi open set (№-SO set) G such that x∈G⊂№cl(G)⊂A_N.

№-SO(U, X), №-θO(U, X) and №-θSO(U, X) respectively denote the families of all nano semi-open(№-SO), nano θ-open(№-θO) and nano θ semi-open(№-θSO)subsets of U.

Definition 2.7. [3] Let (U,$\tau_{\mathcal{R}}$(X)) is a №TS and A_N ⊆ U. Then A_N is said to be nano- bopen set (№- bo- set) if A_N ⊆ №-cl(№-int(A_N)) ∪ №-int(№-cl(A_N)). The complement of nano- b-open set is called nano- b-closed set (№ b$\mathbb{C}$-set).

Example 2.8. [3] Let U = {p,q,r,s} with U/R ={p}, {r}, {q,s} and X = {p,q}.
Then the nano topology τ_{R}(X)= №^{T}= { U, ∅, {p}, {p,q,s}, {q,s}} and nano b-open sets are U, ∅, {p}, {q}, {s},{p, q},{p,r},{p,s},{q, s},{p,q, r},{p,q, s},{q,r,s}.

Definition 2.9. [10**]** A №TS (U, №^{T}) is referred to as nano locally Indiscrete space if every nano open set (№-OS) is nano closed set.(№-CS).

Definition 2.10. Let ($\tilde{U}_{N,}$ τ_N) and ($\tilde{V}_{N,}$ φ_N) be NTS. A mapping : ($\tilde{U}_{N,}$ τ_N) → ($\tilde{V}_{N,}$ φ_N) is said to be

1.№-continuous (№-cts for short) [9] $\zeta^{-1}(Z_N)$ is №-OS in $\tilde{U}_N$ for every №-OS Z_N in $\tilde{V}_N$.

2.№-α-continuous (№-α-cts for short) [14] $\zeta^{-1}(Z_N)$ is №-α-OS in $\tilde{U}_N$ for every №-OS Z_N in $\tilde{V}_N$.

3.№-semi-continuous (№-$\mathcal{S}$-cts for short) [9] $\zeta^{-1}(Z_N)$ is №-$\mathcal{S}$-OS in $\tilde{U}_N$ for every №-OS Z_N in $\tilde{V}_N$.

4.№-pre-continuous (№-$\mathcal{P}$-cts for short) [9] $\zeta^{-1}(Z_N)$ is № $\mathcal{P}$-OS in $\tilde{U}_N$ for every №-OS Z_N in $\tilde{V}_N$.

5.№-b-continuous (№-b-cts for short) [4] $\zeta^{-1}(Z_N)$ is №-b-OS in $\tilde{U}_N$ for every №-OS Z_N in $\tilde{V}_N$.

6.№- θ-continuous (№-θ-cts for short) [3] $\zeta^{-1}(Z_N)$ is №-θ-OS in $\tilde{U}_N$ for every №-OS Z_N in $\tilde{V}_N$.

Definition 2.11. [13] A nano subset A_N of a nano topological space ($\tilde{U}_N$, τ_N) is called nano b$\mathbb{C}$ - open set (№-b$\mathbb{C}$- OS) if for every $\varkappa \in A_N \in$ №-BO($\tilde{U}_N$, τ_N), there exists a nano closed set (№-CS) $\mathcal{H}_N$ such that $\varkappa \in \mathcal{H}_N \subset A_N$.

The family of all nano b$\mathbb{C}$-open sets of a Nano topological space (NTS in short) ($\tilde{U}_N$, τ_N) is denoted by №-B$\mathbb{C}$O($\tilde{U}_N$, τ_N).

Example 2.12 [13] Let $\tilde{U}_N =\{\omega_1, \omega_2, \omega_3, \omega_4\}$,$\tilde{U}_N / \mathcal{R} =\{\{\omega_1\},\{\omega_3\},\{\omega_2, \omega_4\}\}$ and $X_N =\{\omega_1, \omega_2\} \subset \tilde{U}_N$. Then the Nano topology $№^T = \tau_{\mathcal{R}}(X_N)= \{\tilde{U}_N, \emptyset_N, \{\omega_1\},\{\omega_1, \omega_2, \omega_4\},\{\omega_2, \omega_4\}\}$. Then the nano Closed sets are $\tilde{U}_N$, $\emptyset_N$, $\{\omega_2, \omega_3, \omega_4\},\{\omega_3\}$ and $\{\omega_1, \omega_3\}$. Then the collection of all №-b-open sets are №-bO($\tilde{U}_N$, X_N) = $\{\tilde{U}_N, \emptyset_N, \{\omega_1\},\{\omega_2\},\{\omega_4\},\{\omega_1, \omega_2\},\{\omega_1, \omega_3\}, \{\omega_1, \omega_4\},\{\omega_2, \omega_4\},\{\omega_1, \omega_2, \omega_3\},\{\omega_1, \omega_2, \omega_4\},\{\omega_2, \omega_3, \omega_4\}\}$ and №-b$\mathbb{C}$O($\tilde{U}_N$, X_N) = $\{\tilde{U}_N, \emptyset_N, \{\omega_2, \omega_3, \omega_4\},\{\omega_1, \omega_3\}\}$.

3. CONTRA NANO b$\mathbb{C}$- CONTINUOUS FUNCTION

Definition 3.1. Let ($\tilde{U}_{N,}$ τ_N) and ($\tilde{V}_{N,}$ φ_N) be any two NTS. A function : ($\tilde{U}_{N,}$ τ_N) → ($\tilde{V}_{N,}$ φ_N) is said to be contra №-b$\mathbb{C}$-cts if the inverse image of every №-OS in ($\tilde{V}_{N,}$ φ_N) is №-b$\mathbb{C}$-CS in ($\tilde{U}_{N,}$ τ_N).

Example 3.2. Let $\tilde{U}_N = \{\omega_1, \omega_2, \omega_3, \omega_4\}$, $\tilde{U}_N / \mathcal{R} = \{\{\omega_1\}, \{\omega_3\}, \{\omega_2, \omega_4\}\}$ and $X_N = \{\omega_1, \omega_2\} \subset \tilde{U}_N$ and NT = τ_N (X_N) $= \{\tilde{U}_N, \emptyset_N, \{\omega_1\}, \{\omega_2, \omega_4\}, \{\omega_1, \omega_2, \omega_4\}\}$. Then №-b𝕔-CS ($\tilde{U}_N, \tau_N$) $= \{\tilde{U}_N, \emptyset_N, \{\omega_3\}, \{\omega_1, \omega_3\}, \{\omega_2, \omega_3\}, \{\omega_3, \omega_4\}, \{\omega_1, \omega_2, \omega_3\}, \{\omega_1, \omega_3, \omega_4\}, \{\omega_2, \omega_3, \omega_4\}\}$. Let $\tilde{V}_N = \{\tilde{e}_1, \tilde{e}_2, \tilde{e}_3, \tilde{e}_4\}$, $\tilde{V}_N / \mathcal{R} = \{\{\tilde{e}_1, \tilde{e}_2\}, \{\tilde{e}_3, \tilde{e}_4\}\}$ and $K_N = \{\tilde{e}_1, \tilde{e}_2\} \subset \tilde{V}_N$. Then φ_N (K_N) $= \{\tilde{V}_N, \emptyset_N, \{\tilde{e}_1, \tilde{e}_2\}\}$. Define: ($\tilde{U}_N, \tau_N$) → ($\tilde{V}_N, \varphi_N$) by $\zeta(\omega_1) = \tilde{e}_4$, $\zeta(\omega_2) = \tilde{e}_3$, $\zeta(\omega_3) = \tilde{e}_1$, $\zeta(\omega_4) = \tilde{e}_2$. Thus the inverse image of $\{\tilde{e}_1, \tilde{e}_2\}$ in($\tilde{V}_N, \varphi_N$) , i.e, $\zeta^{-1}\{\tilde{e}_1, \tilde{e}_2\} = \{\omega_3, \omega_4\}$ is №-b𝕔-CS in ($\tilde{U}_N, \tau_N$). Hence ζ is contra №-b𝕔- cts.

Theorem 3.3. Let ($\tilde{U}_N, \tau_N$) and ($\tilde{V}_N, \varphi_N$) be any two NTS. A function : ($\tilde{U}_N, \tau_N$) → ($\tilde{V}_N, \varphi_N$) is said to be contra №-b𝕔-cts if and only if the inverse image of every №-CS in ($\tilde{V}_N, \varphi_N$) is №-b𝕔- OS in ($\tilde{U}_N, \tau_N$).

Proof. Let: ($\tilde{U}_N, \tau_N$) → ($\tilde{V}_N, \varphi_N$) be said to be contra №-b𝕔-cts and H_N be any №-CS in ($\tilde{V}_N, \varphi_N$). Since ζ is contra №-b𝕔-cts function, $\zeta^{-1}(\tilde{V}_N - H_N) = \tilde{U}_N - \zeta^{-1}(H_N)$ is №-b𝕔-CS in ($\tilde{U}_N, \tau_N$). Hence $\zeta^{-1}(H_N)$ is №-b𝕔-OS in ($\tilde{U}_N, \tau_N$). Conversely, let G_N be a №-OS in ($\tilde{V}_N, \varphi_N$). By assumption, $\zeta^{-1}(\tilde{V}_N - G_N)$ is №-b𝕔-OS. $\zeta^{-1}(\tilde{V}_N - G_N) = \tilde{U}_N - \zeta^{-1}(G_N)$, $\zeta^{-1}(G_N)$ is №-b𝕔-CS in ($\tilde{U}_N, \tau_N$). Hence ζ is contra №-b𝕔-cts function.

Theorem 3.4. Let: ($\tilde{U}_N, \tau_N$) → ($\tilde{V}_N, \varphi_N$) be a function. If ζ is contra №-θ-cts function then it is contra №-b𝕔-cts function.

Proof. Let: ($\tilde{U}_N, \tau_N$) → ($\tilde{V}_N, \varphi_N$) be a contra №-θ-cts function and H_N be any №-OS in ($\tilde{V}_N, \varphi_N$). Since ζ is contra №-θ-cts function, $\zeta^{-1}(H_N)$ is №-θ-CS in ($\tilde{U}_N, \tau_N$). We know that every №-θ-CS is a №-b𝕔-CS. Therefore $\zeta^{-1}(H_N)$ is №-b𝕔-CS in ($\tilde{U}_N, \tau_N$). Hence ζ is contra №-b𝕔-cts function.

Remark 3.5. Reverse implication of the above theorem need not be true as shown in the following example.

Example 3.6. Let $\tilde{U}_N = \{\omega_1, \omega_2, \omega_3, \omega_4\}$, $\tilde{U}_N / \mathcal{R} = \{\{\omega_1\},\{\omega_3\},\{\omega_2, \omega_4\}\}$, $X_N = \{\omega_1, \omega_2\}$, $\tilde{V}_N = \{\tilde{e}_1, \tilde{e}_2, \tilde{e}_3, \tilde{e}_4\}$, $\tilde{V}_N / \mathcal{R} = \{\{\tilde{e}_2\}, \{\tilde{e}_1, \tilde{e}_3\},\{\tilde{e}_4\}\}$, $\tau_N = \{\tilde{U}_N, E_N, \emptyset_N\}$ and $\varphi_N = \{\tilde{U}_N, F_N, \emptyset_N\}$,where $E_N = \{\{\omega_1\},\{\omega_1, \omega_2, \omega_4\},\{\omega_2, \omega_4\}\}$, $F_N = \{\{\tilde{e}_4\},\{\tilde{e}_1, \tilde{e}_3, \tilde{e}_4\},\{\tilde{e}_1, \tilde{e}_3\}\}$. Then τ_N and φ_N are NTS. №-θ-OS ($\tilde{U}_N, \tau_N$) = $\{\tilde{U}_N, \emptyset_N, \{\omega_2, \omega_4\}\}$.№-$\theta$-CS ($\tilde{U}_N, \tau_N$) = $\{\tilde{U}_N, \emptyset_N, \omega_1, \omega_3\}$. №-b𝕔-CS ($\tilde{U}_N, \tau_N$) = $\{\tilde{U}_N, \emptyset_N, \{\omega_3\},\{\omega_2, \omega_3, \omega_4\}, \{\omega_1, \omega_3\}\}$. №-b𝕔-CS ($\tilde{V}_N, \varphi_N$) = $\{\tilde{U}_N, \emptyset_N, \{\tilde{e}_2\},\{\tilde{e}_1, \tilde{e}_3, \tilde{e}_4\}, \{\tilde{e}_3, \tilde{e}_4\}\}$.

Define ζ: ($\tilde{U}_{N,}$ τ_N) → ($\tilde{V}_{N,}$ φ_N) by ζ(ɷ$_1$) = ẽ$_4$, ζ(ɷ$_2$) = ẽ$_3$, ζ(ɷ$_3$) = ẽ$_2$, ζ(ɷ$_4$) = ẽ1. Then ζ is Contra №-bℂ-cts mapping but not contra №-θcts mapping.

Theorem 3.7. Let : ($\tilde{U}_{N,}$ τ_N) → ($\tilde{V}_{N,}$ φ_N) be a function. If ζ is contra №-θs-cts function then it is contra №-bℂ-cts function.

Proof. Let : ($\tilde{U}_{N,}$ τ_N) → ($\tilde{V}_{N,}$ φ_N) be a contra №-θs-cts function and H_N be any №-OS in ($\tilde{V}_{N,}$ φ_N). Since ζ is contra №-θs-cts function , ζ^{-1}(H_N) is №-θs-CS in ($\tilde{U}_{N,}$ τ_N). We know that every №-θs-CS is a №-bℂ-CS. Therefore ζ^{-1}(H_N) is №-bℂ-CS in ($\tilde{U}_{N,}$ τ_N). Hence ζ is contra №-bℂ-cts function.

Remark 3.8. Reverse implication of the above theorem need not be true as shown in the following example.

Example 3.9. Let $\tilde{U}_N$ ={ɷ$_1$, ɷ$_2$, ɷ$_3$, ɷ$_4$},$\tilde{U}_N$ / $\mathcal{R}$ ={{ɷ$_1$},{ɷ$_{2,}$ ɷ$_4$},{ɷ$_3$}} and X_N ={ɷ$_1$, ɷ$_2$} ⊂ $\tilde{U}_N$. Then the NT = τ_N (X_N) ={$\tilde{U}_N$, ∅$_N$, {ɷ$_1$},{ɷ$_1$, ɷ$_2$, ɷ$_4$},{ɷ$_2$, ɷ$_4$}}. №-bℂ-CS ($\tilde{U}_{N,}$ τ_N) = {$\tilde{U}_N$, ∅$_N$, {ɷ$_3$},{ɷ$_2$, ɷ$_3$, ɷ$_4$}, {ɷ$_1$, ɷ$_3$}}. The №-θS-CS ($\tilde{U}_{N,}$ τ_N) = { $\tilde{U}_N$, ∅$_N$, {ɷ$_1$},{ɷ$_1$, ɷ$_3$},{ɷ$_2$, ɷ$_4$},{ɷ$_1$, ɷ$_2$, ɷ$_4$}}. Let $\tilde{V}_N$ ={ẽ$_1$, ẽ$_2$, ẽ$_3$, ẽ$_4$}, $\tilde{V}_N$ / $\mathcal{R}$ = {{ẽ$_2$}, {ẽ$_{1,}$ ẽ$_4$},{ẽ$_3$}} and K_N ={ẽ$_1$, ẽ$_2$} ⊂ $\tilde{V}_N$ .Then φ_N (K_N) ={ $\tilde{V}_N$, ∅$_N$, {ẽ$_2$},{ẽ$_1$, ẽ$_2$, ẽ$_4$},{ẽ$_1$, ẽ$_3$}}. №-CS ($\tilde{V}_{N,}$ φ_N) ={ $\tilde{V}_N$, ∅$_N$, {ẽ$_3$},{ẽ$_1$, ẽ$_3$, ẽ$_4$},{ẽ$_2$, ẽ$_4$}}. Then τ_N (X_N) and φ_N (K_N) are NT_S. Define: ($\tilde{U}_{N,}$ τ_N) → ($\tilde{V}_{N,}$ φ_N) by ζ(ɷ$_1$) = ẽ$_{3,}$ ζ(ɷ$_2$) = ẽ$_{1,}$ ζ(ɷ$_3$) = ẽ$_{3,}$ ζ(ɷ$_4$) = ẽ$_4$. Here ζ is contra №-bℂ-cts but not contra №-θs- cts mapping.

Result 3.10. The composition of two contra №-bℂ-cts functions need not be contra №-bℂ-cts function as seen from the following example.

Example 3.11. Let $\tilde{U}_N$ ={ɷ$_1$, ɷ$_2$, ɷ$_3$, ɷ$_4$},$\tilde{U}_N$ / $\mathcal{R}$ ={{ɷ$_1$},{ɷ$_{2,}$ ɷ$_4$},{ɷ$_3$}} and X_N ={ɷ$_1$, ɷ$_2$} ⊂ $\tilde{U}_N$ Then the NT = τ_N (X_N) ={$\tilde{U}_N$, ∅$_N$, {ɷ$_1$},{ɷ$_1$, ɷ$_2$, ɷ$_4$},{ɷ$_2$, ɷ$_4$}}. №-bℂ-CS ($\tilde{U}_{N,}$ τ_N) = {$\tilde{U}_N$, ∅$_N$, {ɷ$_3$},{ɷ$_2$, ɷ$_3$, ɷ$_4$}, {ɷ$_1$, ɷ$_3$}}. Let $\tilde{V}_N$ ={ẽ$_1$, ẽ$_2$, ẽ$_3$, ẽ$_4$}, $\tilde{V}_N$ / $\mathcal{R}$ = {{ẽ$_{1,}$ ẽ$_2$},{ẽ$_{3,}$ ẽ$_4$}} and K_N ={ẽ$_1$, ẽ$_2$} ⊂ $\tilde{V}_N$ .Then φ_N (K_N) ={ $\tilde{V}_N$, ∅$_N$, {ẽ$_1$, ẽ$_2$}}. №-bℂ-CS ($\tilde{V}_{N,}$ φ_N) ={ $\tilde{V}_N$, ∅$_N$, {ẽ$_3$}, {ẽ$_4$}, {ẽ$_3$, ẽ$_4$}}.. Define : ($\tilde{U}_{N,}$ τ_N) → ($\tilde{V}_{N,}$ φ_N) by ζ(ɷ$_1$) = ẽ$_{3,}$ ζ(ɷ$_2$) = ẽ$_{1,}$ ζ(ɷ$_3$) = ẽ$_{3,}$ ζ(ɷ$_4$) = ẽ$_4$ .Thus the inverse image {ẽ$_1$, ẽ$_2$} in ($\tilde{V}_{N,}$ φ_N) is №-bℂ-CS in ($\tilde{U}_{N,}$ τ_N). Thus ζ is contra №-bℂ-cts . Let $\dot{W}_N$ ={ δ_1, δ_2, δ_3, δ_4 } and $\dot{W}_N$ / $\mathcal{R}$ = {{ δ_2 }, { δ_3 }, { δ_1, δ_4}}. Let S_N ={ δ_1, δ_3} ⊂ $\tilde{V}_N$ and $\hat{Z}_N$

$(S_N) = \{ \tilde{V}_N, \emptyset_N, \{\delta_3\}, \{\delta_1, \delta_4\}, \{\delta_1, \delta_3, \delta_4\}\}$. Let $\eta : \tilde{V}_N \to \dot{W}_N$ defined by $\eta(\tilde{e}_1) = \delta_4$, $\eta(\tilde{e}_2) = \delta_1$, $\eta(\tilde{e}_1) = \delta_2$, $\eta(\tilde{e}_1) = \delta_3$. Then η is contra №-bc-cts. Clearly these two functions are contra №-bc-cts. But their composition is not contra №-bc-cts since for the №-OS $\{\delta_3\}$ in $(\dot{W}_N, \hat{Z}_N)$, $(\eta \circ \zeta)^{-1} = \zeta^{-1}(\eta^{-1}(\delta_3)) = \{\omega_1\}$ is not №-bc-CS in $(\tilde{U}_N, \tau_N)$. Hence $\eta \circ \zeta$ is not contra №-bc-cts. Thus the composition of two contra №-bc-cts functions need not be contra №-bc-cts function.

Result 3.12. Contra №-bc-cts and №-bc-cts are independent.

Example 3.13. Let $\tilde{U}_N = \{\omega_1, \omega_2, \omega_3, \omega_4\}$, $\tilde{U}_N / \mathcal{R} = \{\{\omega_1, \omega_2\}, \{\omega_3, \omega_4\}\}$ and $X_N = \{\omega_1, \omega_2\} \subset \tilde{U}_N$. Then the NT = $\tau_N(X_N) = \{\tilde{U}_N, \emptyset_N, \{\omega_1, \omega_2\}\}$. №-bc-CS $(\tilde{U}_N, \tau_N) = \{\tilde{U}_N, \emptyset_N, \{\omega_3, \omega_4\}, \{\omega_2, \omega_4\}, \{\omega_1, \omega_3\}\}$. Let $\tilde{V}_N = \{\tilde{e}_1, \tilde{e}_2, \tilde{e}_3, \tilde{e}_4\}$, $\tilde{V}_N / \mathcal{R} = \{\{\tilde{e}_3\}, \{\tilde{e}_2\}, \{\tilde{e}_1, \tilde{e}_4\}\}$ and $K_N = \{\tilde{e}_1, \tilde{e}_3\} \subset \tilde{V}_N$. Then $\varphi_N(K_N) = \{\tilde{V}_N, \emptyset_N, \{\tilde{e}_3\}, \{\tilde{e}_1, \tilde{e}_4\}, \{\tilde{e}_1, \tilde{e}_3, \tilde{e}_4\}\}$. №-CS $(\tilde{V}_N, \varphi_N) = \{\tilde{V}_N, \emptyset_N, \{\tilde{e}_2\}, \{\tilde{e}_2, \tilde{e}_3\}$, Since $\zeta^{-1}(\{\tilde{e}_3, \tilde{e}_4\}) = \{\omega_1, \omega_3\}$. Then ζ is №-bc-CS. But $\zeta^{-1}(\{\tilde{e}_3\}) = \omega_1$ is not №-bc-CS in $(\tilde{U}_N, \tau_N)$. Thus ζ is not contra №-bc-cts.

Example 3.14. Let $\tilde{U}_N = \{\omega_1, \omega_2, \omega_3, \omega_4\}$, $\tilde{U}_N / \mathcal{R} = \{\{\omega_1\}, \{\omega_3\}, \{\omega_2, \omega_4\}\}$ and $X_N = \{\omega_1, \omega_2\} \subset \tilde{U}_N$. Then the NT = $\tau_N(X_N) = \{\tilde{U}_N, \emptyset_N, \{\omega_1\}, \{\omega_2, \omega_4\}, \{\omega_1, \omega_2, \omega_4\}\}$. №-bc-CS $(\tilde{U}_N, \tau_N) = \{\tilde{U}_N, \emptyset_N, \{\omega_2, \omega_3, \omega_4\}, \{\omega_1, \omega_3\}, \{\omega_3\}\}$. Let $\tilde{V}_N = \{\tilde{e}_1, \tilde{e}_2, \tilde{e}_3, \tilde{e}_4\}$, $\tilde{V}_N / \mathcal{R} = \{\{\tilde{e}_1, \tilde{e}_2\}, \{\tilde{e}_3, \tilde{e}_4\}\}$ and $K_N = \{\tilde{e}_1, \tilde{e}_2\} \subset \tilde{V}_N$. Then $\varphi_N(K_N) = \{\tilde{V}_N, \emptyset_N, \{\tilde{e}_1, \tilde{e}_2\}\}$. №-CS $(\tilde{V}_N, \varphi_N) = \{\tilde{V}_N, \emptyset_N, \{\tilde{e}_3, \tilde{e}_4\}\}$. Define : $(\tilde{U}_N, \tau_N) \to (\tilde{V}_N, \varphi_N)$ by $\zeta(\omega_1) = \tilde{e}_4$, $\zeta(\omega_2) = \tilde{e}_3$, $\zeta(\omega_3) = \tilde{e}_1$, $\zeta(\omega_4) = \tilde{e}_2$. Since $\zeta^{-1}(\{\tilde{e}_1, \tilde{e}_2\}) = \{\omega_3, \omega_4\}$ which is №-bc-CS in $(\tilde{U}_N, \tau_N)$. Hence ζ is contra №-bc-CS . But $\zeta^{-1}(\{\tilde{e}_3, \tilde{e}_4\}) = \{\omega_1, \omega_2\}$ is not №-bc-CS in $(\tilde{U}_N, \tau_N)$. Thus ζ is not №-bc-cts.

Theorem 3.15. Let : $(\tilde{U}_N, \tau_N) \to (\tilde{V}_N, \varphi_N)$ be a contra №-bc-cts function. Let $\eta : (\tilde{V}_N, \varphi_N) \to (\dot{W}_N, \xi_N)$ be a №-cts function. Then $\eta \circ \zeta : (\tilde{U}_N, \tau_N) \to (\dot{W}_N, \xi_N)$ is a contra №-bc-cts function.

Proof. Let W_N be a №-OS in $(\dot{W}_N, \xi_N)$. By hypothesis, $\eta^{-1}(W_N)$ is a №-OS in $(\tilde{V}_N, \varphi_N)$. Since ζ is contra №-bc-cts function, $\zeta^{-1}(\eta^{-1}(W_N))$ is a №-bc-CS in $(\tilde{U}_N, \tau_N)$. Hence $\eta \circ \zeta$ is contra №-bc-cts function.

Theorem 3.16. Let : $(\tilde{U}_N, \tau_N) \to (\tilde{V}_N, \varphi_N)$ be a contra №-b𝕔-cts function. Let $\eta : (\tilde{V}_N, \varphi_N) \to (\dot{W}_N, \xi_N)$ be a contra №-cts function. Then $\eta \circ \zeta : (\tilde{U}_N, \tau_N) \to (\dot{W}_N, \xi_N)$ is a №-b𝕔-cts function.

Proof. Let W_N be a №-OS in $(\dot{W}_N, \xi_N)$. By hypothesis, $\eta^{-1}(W_N)$ is a №-CS in $(\tilde{V}_N, \varphi_N)$. Since ζ is contra №-b𝕔-cts function, $\zeta^{-1}(\eta^{-1}(W_N))$ is a №-b𝕔-OS in $(\tilde{U}_N, \tau_N)$. Hence $\eta \circ \zeta$ is a №-b𝕔-cts function.

Theorem 3.17. Let $\zeta: (\tilde{U}_N, \tau_N) \to (\tilde{V}_N, \varphi_N)$ be a bijective function. Then ζ is a contra №-b𝕔-cts function if №-cl $(\zeta(G_N)) \subseteq \zeta$ (№-b𝕔-int (G_N)) for every №-S G_N in $(\tilde{U}_N, \tau_N)$.

Proof. Let G_N be a №-CS in $(\tilde{V}_N, \varphi_N)$. Then №-cl $(G_N) = G_N$ and $\zeta^{-1}(G_N)$ is a №-S in $(\tilde{U}_N, \tau_N)$. By hypothesis, №-cl $(\zeta(\zeta^{-1}(G_N))) \subseteq \zeta$ (№-b𝕔-int $(\zeta^{-1}(G_N))$). Since ζ is onto, $\zeta(\zeta^{-1}(G_N)) = G_N$. Therefore G_N = №-cl (G_N) = №-cl $(\zeta(\zeta^{-1}(G_N))) \subseteq \zeta$ (№-b𝕔-int $(\zeta^{-1}(G_N))$). Now, $G_N \subseteq \zeta$ (№-b𝕔-int $(\zeta^{-1}(G_N))$) which implies $\zeta^{-1}(G_N) \subseteq \zeta^{-1}(\zeta$ (№-b𝕔-int $(\zeta^{-1}(G_N))))$ = №-b𝕔-int $(\zeta^{-1}(G_N)) \subseteq \zeta^{-1}(G_N)$. Hence $\zeta^{-1}(G_N)$ is a №-b𝕔-OS in $(\tilde{U}_N, \tau_N)$. Thus, ζ is a contra №-b𝕔-cts function.

Theorem 3.18. Let $\zeta : (\tilde{U}_N, \tau_N) \to (\tilde{V}_N, \varphi_N)$ be a function. Suppose that one of the following statements hold

i) ζ(№-b𝕔-cl $(H_N) \subseteq$ №-int $(\zeta(H_N))$ for each №-S H_N in $(\tilde{U}_N, \tau_N)$.

ii) №-b𝕔-cl $(\zeta^{-1}(H_N)) \subseteq \zeta^{-1}$(№-int (H_N)) for each №-S H_N in $(\tilde{V}_N, \varphi_N)$.

iii) ζ^{-1}(№-cl $(H_N)) \subseteq$ №-b𝕔-int $(\zeta^{-1}(H_N))$ for each №-S H_N in $(\tilde{V}_N, \varphi_N)$.

Then ζ is a contra №-b𝕔-cts function.

Proof. i) $\Rightarrow$ ii) Let H_N be a №-S in $(\tilde{V}_N, \varphi_N)$. Then $\zeta^{-1}(H_N)$ is №-S in $(\tilde{U}_N, \tau_N)$. By hypothesis, ζ(№-b𝕔-cl $(\zeta^{-1}(H_N)) \subseteq$ №-int $(\zeta(\zeta^{-1}(H_N)) \subseteq$ №-int (H_N). Now,

№-b𝕔-cl $(\zeta^{-1}(H_N)) \subseteq$ №-int (H_N). Therefore, №-b𝕔-cl $(\zeta^{-1}(H_N)) \subseteq \zeta^{-1}$(№-int (H_N)) for each №-S H_N in $(\tilde{V}_N, \varphi_N)$.

ii) $\Rightarrow$ iii) Let H_N be a №-S in $(\tilde{V}_N, \varphi_N)$. Then $\zeta^{-1}(H_N)$ is №-S in $(\tilde{U}_N, \tau_N)$. By hypothesis, №-b𝕔-cl $(\zeta^{-1}(H_N) \subseteq \zeta^{-1}$(№-int (H_N)). Taking complement on both sides, $(\zeta^{-1}$(№-int $(H_N)))^c$ = (№-b𝕔-cl $(\zeta^{-1}(H_N))^c \Rightarrow \zeta^{-1}$(№-int $(H_N))^c \subseteq$ №-b𝕔-int $(\zeta^{-1}(H_N))^c \Rightarrow \zeta^{-1}$(№-cl $(H_N)^c) \subseteq$ №-b𝕔-int $(\zeta^{-1}(H_N)^c)$.Suppose that iii) holds. Let H_N be a №-CS in $(\tilde{V}_N, \varphi_N)$.Then №-cl $(H_N) = H_N$

and $\zeta^{-1}(H_N)$ is a №-S in $(\tilde{U}_N, \tau_N)$. Now $\zeta^{-1}(H_N) = \zeta^{-1}$(№-cl (H_N)) ⊆ №-bℂ-int $(\zeta^{-1}(H_N))$ ⊆ $\zeta^{-1}(H_N)$. Then $\zeta^{-1}(H_N)$ is a №-bℂ- OS in in $(\tilde{U}_N, \tau_N)$. Hence ζ is a contra №-bℂ-cts function.

Theorem 3.19. Let $\zeta : (\tilde{U}_N, \tau_N) \to (\tilde{V}_N, \varphi_N)$ be any NTS. Let $\zeta : (\tilde{U}_N, \tau_N) \to (\tilde{V}_N, \varphi_N)$ be a function. Suppose that one of the following properties hold.

i) ζ^{-1}(№-bℂ-cl (H_N)) ⊆ №-bℂ-int (№-bℂ-cl $(\zeta^{-1}(H_N))$) for each №-S H_N in $(\tilde{V}_N, \varphi_N)$.

ii) №-bℂ-cl (№-bℂ-int $(\zeta^{-1}(H_N))$) ⊆ ζ^{-1}(№-bℂ-int (H_N)) for each №-S H_N in $(\tilde{V}_N, \varphi_N)$.

iii) ζ(№-bℂ-cl (№-bℂ-int (G_N))) ⊆ №-bℂ-int $(\zeta(G_N))$ for each №-S G_N in $(\tilde{U}_N, \tau_N)$.

iv) ζ (№-cl (H_N)) ⊆ №-bℂ-int $(\zeta(G_N))$ for each №-S G_N in $(\tilde{U}_N, \tau_N)$.

Then ζ is a contra №-bℂ-cts.

Proof. i) ⇒ ii) Let H_N be a №-S in $(\tilde{V}_N, \varphi_N)$. Then $\zeta^{-1}(H_N)$ is a №-S in $(\tilde{U}_N, \tau_N)$. By hypothesis, ζ^{-1}(№-bℂ-cl (H_N)) ⊆ №-bℂ-int (№-bℂ-cl $(\zeta^{-1}(H_N))$).

Taking complement on both sides, (№-bℂ-int (№-bℂ-cl $(\zeta^{-1}(H_N))$))^c ⊆ (ζ^{-1} (№-bℂ-cl (H_N)))^c ⇒ №-bℂ-cl (№-bℂ-cl $(\zeta^{-1}(H_N))$)^c ⊆ ζ^{-1} (№-bℂ-cl (H_N))^c ⇒ №-bℂ-cl (№-bℂ-int $(\zeta^{-1}(H_N))^c$) ⊆ ζ^{-1}(№-bℂ-int $(H_N{}^c)$) ⇒ №-bℂ-cl (№-bℂ-int $(\zeta^{-1}(H_N{}^c))$) ⊆ (№-bℂ-int $(H_N{}^c)$). Hence №-bℂ-cl (№-bℂ-int $(\zeta^{-1}(H_N))$) ⊆ ζ^{-1}(№-bℂ-int (H_N)) for each №-S H_N in $(\tilde{V}_N, \varphi_N)$.

ii) ⇒ iii) Let G_N be a №-S in $(\tilde{U}_N, \tau_N)$. Let $H_N = \zeta(G_N)$, then $G_N \subseteq \zeta^{-1}(H_N)$. By hypothesis,№-bℂ-cl (№-bℂ-int (G_N)) ⊆ №-bℂ-cl (№-bℂ-int $(\zeta^{-1}(H_N))$) ⊆ ζ^{-1} (№-bℂ-int (H_N)) ⇒ №-bℂ-cl (№-bℂ-int (G_N)) ⊆ ζ^{-1} (№-bℂ-int (H_N). Therefore, ζ(№-bℂ-cl (№-bℂ-int (G_N))) ⊆ ζ^{-1} (№- bℂ-int (H_N) = №-bℂ-int $(\zeta(G_N))$. Hence ζ(№-bℂ-cl (№-bℂ-int (G_N))) ⊆ №-bℂ-int $(\zeta(G_N))$.

iii) ⇒ iv) Let G_N be a №-S in $(\tilde{U}_N, \tau_N)$.Then №-bℂ-int $(G_N) = G_N$. By hypothesis, ζ(№-bℂ--cl (G_N)) = ζ(№-bℂ-cl (№-bℂ-int $(\zeta(G_N))$) ⊆ №-bℂ-int $(\zeta(G_N))$.Thus, ζ(№-bℂ-cl (G_N)) ⊆ №-bℂ-int $(\zeta(G_N))$.

Suppose (iv) holds. Let H_N be a №-OS in $(\tilde{V}_N, \varphi_N)$. Then $\zeta^{-1}(H_N) = G_N$ is a №-S in $(\tilde{U}_N, \tau_N)$. By hypothesis, ζ(№-bℂ-cl (G_N)) ⊆ №-bℂ-int $(\zeta(G_N))$. Now, ζ(№-bℂ-cl (G_N)) ⊆ №-bℂ-int $(\zeta(G_N)) \subseteq \zeta(G_N)$ ⇒ ζ(№-bℂ-cl (G_N)) ⊆ $\zeta(G_N)$ ⇒ №-bℂ-cl $(G_N) \subseteq \zeta^{-1}(\zeta(G_N)) = G_N$ .This means that №-b𝕔-cl (G_N) ⊆ G_N . But G_N ⊆ №-b𝕔-cl (G_N). Hence G_N = №-b𝕔-cl (G_N). Hence G_N is a №-b𝕔-CS in ($\tilde{U}_N, \tau_N$). Hence ζ is a contra №-b𝕔-cts.

Theorem 3.20. If ζ : ($\tilde{U}_N, \tau_N$) → ($\tilde{V}_N, \varphi_N$) is contra №-b𝕔-cts and ($\tilde{V}_N, \varphi_N$) is №-regular, then ζ is №-b𝕔-cts.

Proof. Let x ∈ $\tilde{U}_N$ and H_N be any №-OS in ($\tilde{V}_N, \varphi_N$) containing ζ(x). Since ($\tilde{V}_N, \varphi_N$) is №-regular, there exists a №-OS G_N in ($\tilde{V}_N, \varphi_N$) containing ζ(x) such that №-cl (G_N) ⊆ H_N . Since ζ is contra №-b𝕔-cts, there exists a №-b𝕔-OS W_N of ($\tilde{U}_N, \tau_N$) containing x such that ζ(W_N) ⊆ №-cl (G_N). Then ζ(W_N) ⊆ №-cl (G_N) ⊆ H_N . Hence ζ is №-b𝕔-cts.

V.CONCLUSION

Many different forms of continuous functions have been introduced over the years. Its importance is significant in various areas of mathematics and related sciences. In this paper we presented contra №-bc continuous mappings and discussed some of their properties .

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